AN INTRODUCTION TO CLASSICAL COSMOLOGY

I. Observational Foundations of Cosmology

The observations that form the basis of any cosmological theory are fairly simple, but they have profound implications.

1. The sky is *dark*. This simple observation forms the basis for Olber's Paradox. It means that the universe cannot be simultaneously *a)* infinite in space; *b)* infinite in time; and *c)* static. If the universe did satisfy these conditions, then any line-of-sight on the sky would eventually intercept the surface of a star, and the sky would appear to have the brightness of a typical star. Although Olber's Paradox is easily resolved by standard cosmological models, it is worth noting here that it cannot be resolved simply by assuming that the universe is filled uniformly with an absorbing medium such as dust. The reason is that ultimately that medium would heat up and reradiate the absorbed energy at the same rate at which it is absorbed. In fact, the sky is not completely dark - it glows at a temperature of approximately 3 K due to the microwave background radiation.

2. The universe is expanding, and locally this expansion is linear: $v_r = H_0 r$, where v_r is the observed radial velocity of a galaxy, r is its distance, and H_0 is a proportionality constant (the Hubble constant). Demonstrating the linearity of this law with high precision turns out to be surprisingly difficult. The biggest problem is establishing a class of objects that can be used as standard candles (*i.e.*, that have a known luminosity). A second problem is that galaxies have large peculiar motions superposed on the pattern of uniform expansion. For example, the Milky Way has a peculiar motion relative to the microwave background radiation rest frame of over 600 km/s. (reference??)

3. The universe is isotropic. Classically, this result is demonstrated by comparing the density of extragalactic objects such as galaxies, radio sources, etc. in different directions. Two effects that hamper these measurements are foreground extinction due to dust in our own galaxy and the inherent clustering of galaxies that introduce additional statistical fluctuations beyond Poisson statistics. The best demonstration of isotropy comes from the 3 K background radiation, which (aside from the peculiar motion of the Milky Way relative to the microwave rest frame) is isotropic at a level of $< 10^{-4}$ on all angular scales.

4. The universe is homogeneous. Tests for homogeneity rely on showing that the distribution of galaxies is independent of distance. In practice, this is by done using galaxy counts. If one counts galaxies out to a distance D, then the number of galaxies that will be counted is $N \propto D^3$. In practice, one counts galaxies to a limiting apparent brightness and looks at how the number varies as a

function of that brightness. The scaling law takes the form $\log_{10} N = 0.6m +$ constant, where m is the limiting apparent magnitude. This law holds even when observing an ensemble of galaxies with a distribution of intrinsic luminosities. Observations show that this law does hold but only in the magnitude range $12 < m_B < 16 - 17$. At brighter magnitudes, large scale clustering (especially due to the Virgo cluster) dominates. At fainter limiting magnitudes, one is observing galaxies with a large enough velocity that the law needs to be modified to account for:

a) a decrease in the apparent brightness of a galaxy due to its recession velocity (the so-called K correction),
b) time evolution of the galaxy luminosity, and
c) time evolution of the geometry of the universe.

Although the conditions of isotropy and homogeneity have a similar character, they are not identical. One can have an isotropic universe that is not homogeneous (*e.g.*. if Hubble's law $v = H_0 r$ is replaced by $v \propto r^2$). However, it can be shown that a universe that is isotropic for all observers must be homogeneous. Conversely, a homogeous universe need not be isotropic. The most general expansion law that a homogeneous universe can have is $\vec{v} = H_0\vec{r} + \vec{\Omega} \times \vec{r}$, where $\vec{\Omega}$ is a constant vector. To demonstrate isotropy, consider the velocity measured by an observer at a position $\vec{r'} = \vec{r} + \vec{\delta}$. That observer measures a velocity $\vec{v'} = \vec{v} - \vec{v_\delta}$, where $\vec{v_\delta}$ is the velocity of the second observer relative to the first: $\vec{v_\delta} = H_0\vec{\delta} + \vec{\Omega} \times \vec{\delta}$. Combining, we find that $\vec{v'} = H_0\vec{r'} + \vec{\Omega} \times \vec{r'}$, and so the form of the expansion law is unchanged. Although it is homogenous, such a universe is not isotropic - there is a preferred direction that could be detected, e.g., via a quadrupole moment in the microwave background that is not seen.

The observations of isotropy and homogeneity are so fundamental to cosmological models that they have been cast into a fundamental tenet known at the Cosmological Principle. In plain language, the Cosmological Principle holds that their is no special place in the universe: any observer in the universe should have a view that is essentially indistinguishable from that of any other observer. The exact sense of what is meant by an observer will be refined in Chapter III. The Cosmological Principle severely limits the number of possible cosomological models.

The observed expansion of the universe means that in the past galaxies were closer together. If we run the clock backwards, we find that all objects started expanding at the same time. This event is called the Big Bang.

It is a bit curious that even though the observations of isotropy and homogeneity are fundamental to standard cosmological models, they have never been tested exhaustively (although the are known to be true approximately), and so one might ask why this is so. One reason is that relativistic cosmologies were being developed at a time when the Milky Way was thought to constitute the entirety of the observable universe, and the distribution of stars in the Milky Way is certainly not homogeneous or isotropic. The distribution of "nebulae" was known to be more isotropic than that of stars, and so when their extragalactic nature was recognized (at least of those that we now call galaxies), it was straightforward to conclude that the distribution of galaxies (and hence the observable universe) is uniform.